Cuaderno de notas de trabajo

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Cuaderno Sn 19

La Teoría de Birkhoff en Métrica Euclidiana:

1) $ds^2 = dt^2 dx^2 + dy^2 + dz^2$. ds^2 de la métrica euclidiana

2) $ds^2 = dT^2 - dX^2 - dY^2 - dZ^2$. ds^2 de la métrica pseudoeuclidiana

3) La ecuación (1) se puede escribir:

$$ds^2 = \Delta_{ij}\, dx^i dx^j .$$

4) La ecuación (2) se puede escribir

$$ds^2 = \delta_{ij}\, dX^i dX^j .$$

5) Ecuaciones de transformación:

$T = t$	$t = T$
$X = -i x$	$x = i X$
$Y = -i y$	$y = i Y$
$Z = -i z$	$z = i Z$.

6) Las matrices de las transformaciones son:
~~directa:~~

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & i & 0 & 0 \\ 0 & 0 & i & 0 \\ 0 & 0 & 0 & i \end{pmatrix} = \frac{\partial x^i}{\partial \bar{X}^j}$$

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -i & 0 & 0 \\ 0 & 0 & -i & 0 \\ 0 & 0 & 0 & -i \end{pmatrix} = \frac{\partial \bar{X}^i}{\partial x^j}$$

7) El tensor potencial gravitacional doblemente covariante es:

Métrica pseudoeuclidiana	Métrica euclidiana
H_{ij}	h_{ij}

8) $$h_{ij} = \frac{\partial X^m}{\partial x^i} \frac{\partial X^n}{\partial x^j} H_{mn}$$

9) Los índices griegos $\alpha, \beta, \gamma, \delta, \ldots$ van de 2 a 4

Los índices latinos van de 1 a 4.

10) $$h_{11} = \frac{\partial X^m}{\partial x^1} \frac{\partial X^n}{\partial x^1} H_{mn}$$

$$\boxed{h_{11} = H_{11}}$$

$$h_{1\alpha} = \frac{\partial X^m}{\partial x^1} \cdot \frac{\partial X^n}{\partial x^\alpha} H_{mn}$$

$$\boxed{h_{1\alpha} = i\, H_{1\alpha}}$$

$$\boxed{h_{\alpha 1} = i H_{\alpha 1}}$$

$$h_{\alpha\beta} = \frac{\partial X^m}{\partial x^\alpha}\frac{\partial X^n}{\partial x^\beta} H_{mn}$$

$$\boxed{h_{\alpha\beta} = - H_{\alpha\beta}}$$

$$\begin{pmatrix} h_{11} & h_{12} & h_{13} & h_{14} \\ h_{21} & h_{22} & h_{23} & h_{24} \\ h_{31} & h_{32} & h_{33} & h_{34} \\ h_{41} & h_{42} & h_{43} & h_{44} \end{pmatrix} = \begin{pmatrix} H_{11} & iH_{12} & iH_{13} & iH_{14} \\ iH_{21} & -H_{22} & -H_{23} & -H_{24} \\ iH_{31} & -H_{32} & -H_{33} & -H_{34} \\ iH_{41} & -H_{42} & -H_{43} & -H_{44} \end{pmatrix}$$

$$\begin{pmatrix} H_{11} & H_{12} & H_{13} & H_{14} \\ H_{21} & H_{22} & H_{23} & H_{24} \\ H_{31} & H_{32} & H_{33} & H_{34} \\ H_{41} & H_{42} & H_{43} & H_{44} \end{pmatrix} = \begin{pmatrix} h_{11} & -ih_{12} & -ih_{13} & -ih_{14} \\ -ih_{21} & -h_{22} & -h_{23} & -h_{24} \\ -ih_{31} & -h_{32} & -h_{33} & -h_{34} \\ -ih_{41} & -h_{42} & -h_{43} & -h_{44} \end{pmatrix}$$

11) Ecuaciones diferenciales de la línea de universo de una partícula exploradora de un campo gravitacional.

Métrica pseudoeuclidiana

$$\frac{d^2X^i}{ds^2} = \Delta^{im}\left(\frac{\partial H_{mj}}{\partial X^k} - \frac{\partial H_{jk}}{\partial X^m}\right)\frac{dX^j}{ds}\frac{dX^k}{ds}$$

Métrica euclidiana

$$\frac{d^2x^i}{ds^2} = \delta^{im}\left(\frac{\partial h_{mj}}{\partial x^k} - \frac{\partial h_{jk}}{\partial x^m}\right)\frac{dx^j}{ds}\frac{dx^k}{ds}$$

$$\frac{d^2x^i}{ds^2} = \left(\frac{\partial h_{ij}}{\partial x^k} - \frac{\partial h_{jk}}{\partial x^i}\right)\frac{dx^j}{ds}\frac{dx^k}{ds}$$

Las Fórmulas de Serret–Frenet en 4 dimensiones.

Sea τ^i el cuadrivector tangente a una curva $x^i = x^i(s)$.

$$\boxed{\tau^i = \frac{dx^i}{ds}}$$ (cuadri-) Vector tangente!

Derívese intrínsecamente con respecto a s.

$$\frac{\delta \tau^i}{\delta s} = \tau^i_{\ ,j} \frac{dx^j}{ds}$$

$$\frac{\delta \hat{\tau}}{\delta s} = \kappa_1 \hat{n}_1$$

$\hat{n}_1 = \left\{ \text{cuadrivector unitario normal ~~a la cu~~ principal de la curva} \right.$

$$\left| \frac{\delta \hat{\tau}}{\delta s} \right| = \kappa_1$$

$\hat{\tau}$ = cuadrivector unitario tangente a la curva

$\hat{n}_1$ = cuadrivector unitario normal principal de la curva.

$\hat{n}_2$ = cuadrivector unitario normal a la curva.

$\hat{n}_2 \perp (\hat{\tau}, \hat{n}_1)$

$\hat{n}_2$ está en el espacio de 3 dimens. subtendido por $\hat{\tau}, \frac{\partial \hat{\tau}}{\partial s}, \frac{\partial^2 \hat{\tau}}{\partial s^2}$.

~~E_2 = Espacio euclidiano de dos dimensiones~~

Las Fórmulas de Frenet en 4 dimensiones.

C. E. Weatherburn. An Introduction to Riemannian Geometry and Tensor Calculus. Cambridge University Press. 1942 pág. 95, 96.

~~Alrun~~

$\hat{r}$ — — — cuadrivector de posición de un punto de una curva.

$\hat{t} = \frac{d\hat{r}}{ds}$ — — cuadrivector unitario tangente a la curva

$\hat{n}_1$ — — — — cuadrivector unitario normal principal de la curva.

$$\boxed{\frac{\delta \hat{t}}{\delta s} = \kappa_1 \hat{n}_1}$$

κ_1 — — — — primera curvatura de la curva.

$$\boxed{\text{Considérese } \frac{\delta \hat{n}_1}{\delta s}.}$$

1) $\frac{\delta \hat{n}_1}{\delta s}$ es un cuadrivector que tiene una componente en el espacio bidimensional subtendido por $\hat{t}$ y $\hat{n}_1$ y otra componente $\perp$ a ese espacio.

~~2) por ser $\hat{n}_1$ unitario, $\frac{\delta \hat{n}_1}{\delta s}$ es $\perp$ a $\hat{n}_1$~~

2) $$\frac{\delta \hat{n}_1}{\delta s} = \underbrace{\alpha_1 \hat{t} + \beta_1 \hat{n}_1}_{\substack{\text{Componente} \\ \text{en el espacio} \\ \text{bidimensional} \\ \text{subt. por } \hat{t} \text{ y } \hat{n}_1}} + \underbrace{\kappa_2 \hat{n}_2}_{\substack{\text{Componente} \\ \perp \text{ a } \hat{n}_1 \\ \text{y } \perp \hat{t}}}$$

cuadrivector unitario

3) Por ser $\hat{n}_1$ unitario, $\frac{\delta \hat{n}_1}{\delta s} \perp \hat{n}_1$

y $\beta_1 = 0$

$$\frac{\delta \hat{n}_1}{\delta s} = \alpha_1 \hat{\tau} + \kappa_2 \hat{n}_2$$

κ_2 es la segunda curvatura

$$\hat{\tau} \cdot \hat{n}_1 = 0$$

$$\frac{\delta \hat{\tau}}{\delta s} \cdot \hat{n}_1 + \hat{\tau} \cdot \frac{\delta \hat{n}_1}{\delta s} = 0 \qquad \frac{\delta \hat{\tau}}{\delta s} = \kappa_1 \hat{n}_1$$

$$\kappa_1 + \hat{\tau} \cdot \frac{\delta \hat{n}_1}{\delta s} = 0$$

$$\hat{\tau} \cdot \frac{\delta \hat{n}_1}{\delta s} = -\kappa_1 \qquad \alpha_1 = -\kappa_1$$

$$\boxed{\frac{\delta \hat{n}_1}{\delta s} = -\kappa_1 \hat{\tau} + \kappa_2 \hat{n}_2}$$

segunda curvatura

$\boxed{\text{Considérese } \frac{\delta \hat{n}_2}{\delta s}}$

1) $\frac{\delta \hat{n}_2}{\delta s}$ es un cuadrivector que tiene una componente en el espacio tridimensional subtendido por $\hat{\tau}, \hat{n}_1, \hat{n}_2$ y otra componente $\perp$ a ese espacio.

2) $\frac{\delta \hat{n}_2}{\delta s} = \alpha_2 \hat{\tau} + \beta_2 \hat{n}_1 + \gamma_2 \hat{n}_2 + \kappa_3 \hat{n}_3$

$\hat{n}_3$ cuadrivector unitario $\perp$ a $\hat{\tau}$, $\hat{n}_1$ y $\hat{n}_2$.

κ_3 = tercera curvatura.

3) Como $\hat{n}_2$ es unitario $\frac{\delta \hat{n}_2}{\delta s} \perp n_2$

$\therefore \gamma_2 = 0$

4) $\hat{\tau} \cdot \hat{n}_2 = 0$

$$\frac{\delta \hat{t}}{\delta s} \cdot \hat{n}_2 + \hat{t} \cdot \frac{\delta \hat{n}_2}{\delta s} = 0$$

$$+\kappa_1 \hat{n}_1 \cdot \hat{n}_2 + \hat{t} \cdot \frac{\delta \hat{n}_2}{\delta s} = 0$$

cero

$$\hat{t} \cdot \frac{\delta \hat{n}_2}{\delta s} = 0$$

$$\hat{t} \perp \frac{\delta \hat{n}_2}{\delta s}. \qquad \alpha_2 = 0$$

$$\cancel{\hat{n}_2 \cdot \hat{n}_2 = 0}$$

$$\cancel{\hat{n}_2 \cdot \delta}$$

$$\hat{n}_1 \cdot \hat{n}_2 = 0$$

$$\hat{n}_1 \cdot \frac{\delta \hat{n}_2}{\delta s} + \hat{n}_2 \cdot \frac{\delta \hat{n}_1}{\delta s} = 0$$

$$\hat{n}_2 \cdot \frac{\delta \hat{n}_1}{\delta s} = \kappa_2$$

$$\hat{n}_1 \cdot \frac{\delta \hat{n}_2}{\delta s} = -\kappa_2 \quad \| \quad \gamma_2 = -\kappa_2$$

$$\boxed{\frac{\delta \hat{n}_2}{\delta s} = -\kappa_2 \hat{n}_2 + \kappa_3 \hat{n}_3}$$

Considérese $\boxed{\dfrac{\delta \hat{n}_3}{\delta s}}$

~~$\dfrac{\delta \hat{n}_3}{\delta s}$ es un cuadrivector que una componente en el espacio tridimensional subtendido por $\hat{\tau}, \hat{n}_1, \hat{n}_2$~~

$\dfrac{\delta \hat{n}_3}{\delta s}$ es un cuadrivector que puede expresarse en función de los cuatro cuadrivectores $\hat{\tau}, \hat{n}_1, \hat{n}_2, \hat{n}_3$. Cada uno de estos cuatro cuadrivectores es normal a los otros tres.

$\hat{\tau} \perp (\hat{n}_1, \hat{n}_2\ \hat{n}_3)$;

$\hat{n}_1 \perp (\hat{\tau}, \hat{n}_2, \hat{n}_3)$;

$\hat{n}_2 \perp (\hat{\tau}, \hat{n}_1, \hat{n}_3)$;

$\hat{n}_3 \perp (\hat{\tau}, \hat{n}_1, \hat{n}_2)$,

$$\frac{\delta \hat{n}_3}{\delta s} = \alpha_3 \hat{t} + \beta_3 \hat{n}_1 + \gamma_3 \hat{n}_2 + \varepsilon_3 \hat{n}_3$$

Como $\hat{n}_3$ es unitario,

$$\frac{\delta \hat{n}_3}{\delta s} \perp \hat{n}_3 \quad \therefore \quad \boxed{\varepsilon_3 = 0}$$

$$\hat{t} \cdot \hat{n}_3 = 0$$

$$\frac{\delta \hat{t}}{\delta s} \cdot \hat{n}_3 + \hat{t} \cdot \frac{\delta \hat{n}_3}{\delta s} = 0$$

$$\kappa_1 \hat{n}_1 \cdot \hat{n}_3 + \hat{t} \cdot \frac{\delta \hat{n}_3}{\delta s} = 0$$

$$\therefore \quad \boxed{\alpha_3 = 0}$$

$$\hat{n}_1 \cdot \hat{n}_3 = 0$$

$$\frac{\delta \hat{n}_1}{\delta s} \cdot \hat{n}_3 + \hat{n}_1 \cdot \frac{\delta \hat{n}_3}{\delta s} = 0$$

$$\left[-\kappa_1 \hat{t} + \kappa_2 \hat{n}_2\right] \cdot \hat{n}_3 + \hat{n}_1 \cdot \frac{\delta \hat{n}_3}{\delta s} = 0$$

$$\hat{n}_1 \perp \frac{\delta \hat{n}_3}{\delta s} \qquad \boxed{\beta_3 = 0}$$

$$\hat{n}_2 \cdot \hat{n}_3 = 0$$

$$\frac{\delta \hat{n}_2}{\delta s} \cdot \hat{n}_3 + \hat{n}_2 \cdot \frac{\delta \hat{n}_3}{\delta s} = 0$$

$$\left[-\kappa_2 \hat{n}_2 + \kappa_3 \hat{n}_3 \right] \cdot \hat{n}_3 + \hat{n}_2 \cdot \frac{\delta \hat{n}_3}{\delta s} = 0$$

$$+\kappa_3 + \hat{n}_2 \cdot \frac{\delta \hat{n}_3}{\delta s} = 0$$

$$\hat{n}_2 \cdot \frac{\delta \hat{n}_3}{\delta s} = -\kappa_3$$

$$\therefore \quad \boxed{\int_3 = -\kappa_3}$$

$$\boxed{\frac{\delta \hat{n}_3}{\delta s} = -\kappa_3 \hat{n}_2}$$

Fórmulas de Frenet.

4 dimensiones.

$$\boxed{\hat{\tau} = \frac{d\hat{r}}{ds}.}$$

$$\boxed{\frac{\delta \hat{\tau}}{\delta s} = \kappa_1 \hat{n}_1.}$$

$$\boxed{\frac{\delta \hat{n}_1}{\delta s} = -\kappa_1 \hat{\tau} + \kappa_2 \hat{n}_2}$$

$$\boxed{\frac{\delta \hat{n}_2}{\delta s} = -\kappa_2 \hat{n}_2 + \kappa_3 \hat{n}_3}$$

$$\boxed{\frac{\delta \hat{n}_3}{\delta s} = -\kappa_3 \hat{n}_2}$$

Comprobado pág. 95 del C.E. Weatherburn
An Introduction to Riemannian Geometry
and Tensor Calculus.
Cambridge University Press. 1942

$$[\hat{r}, \hat{\tau}, \hat{n}]$$

$$\left\| \begin{matrix} x^1 & x^2 & x^3 & x^4 \\ \tau^1 & \tau^2 & \tau^3 & \tau^4 \\ n^1_{(1)} & n^2_{(1)} & n^3_{(1)} & n^4_{(1)} \end{matrix} \right\| = \hat{Q}$$

0

$$\frac{\delta \hat{Q}}{\delta s} = \left\| \begin{matrix} \tau^1 & \tau^2 & \tau^3 & \tau^4 \\ \tau^1 & \tau^2 & \tau^3 & \tau^4 \\ n^1_{(1)} & n^2_{(1)} & n^3_{(1)} & n^4_{(1)} \end{matrix} \right\| \cdots$$

0

$$+ \left\| \begin{matrix} x^1 & x^2 & x^3 & x^4 \\ \kappa_1 n^1_{(1)} & \kappa_1 n^2_{(1)} & \kappa_1 n^3_{(1)} & \kappa_1 n^4_{(1)} \\ n^1_{(1)} & n^2_{(1)} & n^3_{(1)} & n^4_{(1)} \end{matrix} \right\|$$

$$+ \left\| \begin{matrix} x^1 & x^2 & x^3 \\ \tau^1 & \tau^2 & \tau^3 \\ -\kappa_1 \tau^1 + \kappa_2 n^1_{(2)} & -\kappa_1 \tau^2 + \kappa_2 n^2_{(2)} & -\kappa_1 \tau^3 + \kappa_2 n^3_{(2)} \end{matrix} \right.$$

Mecánica de Minkowski

$$\hat{v} = \frac{d\hat{r}}{ds} = \frac{\delta\hat{r}}{\delta s} = \hat{\tau}$$

$$\hat{a} = \frac{\delta\hat{v}}{\delta s} = \frac{\delta\hat{\tau}}{\delta s} = \kappa_1 \hat{n}_1$$

$\boxed{a = \kappa_1}$ La aceleración es igual a la primera curvatura.

$$\cancel{\frac{\delta\hat{a}}{\delta s} = -\kappa_1}$$

$$\frac{\delta\hat{a}}{\delta s} = \kappa_1' \hat{n}_1 + \kappa_1\left[-\kappa_1\hat{\tau} + \kappa_2\hat{n}_2\right]$$

$$\frac{\delta\hat{a}}{\delta s} = a'\hat{n}_1 - \kappa_1^2\hat{v} + \kappa_1\kappa_2\hat{n}_2$$

$$\cancel{\frac{\delta\hat{a}}{\delta s} = a'\hat{v}' - a^2\hat{v} + a\kappa_2\hat{n}_2}$$

$$\frac{\delta\hat{a}}{\delta s} = a'\frac{\hat{a}}{a} - a^2\hat{v} + a\kappa_2\hat{n}_2$$

Mecánica de Minkowski

$$\boxed{\hat{v} = \frac{d\hat{r}}{ds} = \frac{\delta \hat{r}}{\delta s} = \hat{\tau}}$$ vector unitario tangente a la curva

$$\hat{a} = \frac{\delta \hat{v}}{\delta s} = \frac{\delta \hat{\tau}}{\delta s} = \kappa_1 \hat{n}_1 = a \hat{n}_1$$

$$a = \kappa_1$$ la aceleración es la primera curvatura

La dirección de la aceleración es la de la normal principal.

$$\frac{\delta \hat{a}}{\delta s} = a' \hat{n}_1 + a\left[-a\hat{v} + \kappa_2 \hat{n}_2\right]$$

$$\boxed{\frac{\delta \hat{a}}{\delta s} = \frac{a'}{a}\hat{a} - a^2 \hat{v} + a\kappa_2 \hat{n}_2}$$

$$\boxed{\hat{v} \cdot \frac{\delta \hat{a}}{\delta s} = -a^2}$$

$$\hat{v} \cdot \hat{a} = 0 \qquad \hat{v} \cdot \frac{\delta \hat{a}}{\delta s} + \hat{a} \cdot \hat{a} = 0$$

$$\hat{a}\cdot\frac{\delta\hat{a}}{\delta s} = a^{i} a^{\prime}$$

$$a^{i} = \left(\frac{\partial h_{ij}}{\partial x^{k}} - \frac{\partial h_{jk}}{\partial x^{i}}\right)\frac{dx^{j}}{ds}\frac{dx^{k}}{ds}$$

$$a^{i} a^{i} = \left[\frac{\partial h_{ij}}{\partial x^{k}}\frac{\partial h_{ip}}{\partial x^{q}}\frac{dx^{j}}{ds}\frac{dx^{k}}{ds}\frac{dx^{p}}{ds}\frac{dx^{q}}{ds} - \frac{\partial h_{jk}}{\partial x^{i}}\frac{\partial h_{pq}}{\partial x^{i}}\frac{dx^{j}}{ds}\frac{dx^{k}}{ds}\frac{dx^{p}}{ds}\frac{dx^{q}}{ds}\right]$$

Campo Central en métrica euclidiana.

$$\begin{pmatrix} H_{11} & H_{12} & H_{13} & H_{14} \\ H_{21} & H_{22} & H_{23} & H_{24} \\ H_{31} & H_{32} & H_{33} & H_{34} \\ H_{41} & H_{42} & H_{43} & H_{44} \end{pmatrix} = \begin{pmatrix} \frac{M}{r} & 0 & 0 & 0 \\ 0 & \frac{M}{r} & 0 & 0 \\ 0 & 0 & \frac{M}{r} & 0 \\ 0 & 0 & \underset{\text{cero}}{\cancel{}} & \frac{M}{r} \end{pmatrix}$$

$$\begin{pmatrix} h_{11} & h_{12} & h_{13} & h_{14} \\ h_{21} & h_{22} & h_{23} & h_{24} \\ h_{31} & h_{32} & h_{33} & h_{34} \\ h_{41} & h_{42} & h_{43} & h_{44} \end{pmatrix} = \begin{pmatrix} \frac{M}{r} & 0 & 0 & 0 \\ 0 & -\frac{M}{r} & 0 & 0 \\ 0 & 0 & -\frac{M}{r} & 0 \\ 0 & 0 & 0 & -\frac{M}{r} \end{pmatrix}$$

$$\boxed{h_{ij} = \frac{M}{r}\Delta_{ij}}$$

campo central
métrica euclidiana

Traza del tensor gravitacional

$$\boxed{h_{ii} = -2\frac{M}{r}}$$

$\alpha = 2, 3, 4$

$$\boxed{\frac{\partial h_{ii}}{\partial x^\alpha} = -i\frac{\partial h_{ii}}{\partial X^\alpha}}$$

$$\frac{\partial}{\partial X^\alpha}\left[-\frac{2M}{r}\right] = -2M\left[-\frac{1}{r^2}\frac{X^\alpha}{r}\right]$$

$$\frac{\partial}{\partial X^\alpha}\left[-\frac{2M}{r}\right] = +\frac{2MX^\alpha}{r^3}$$

$$\frac{\partial}{\partial T}\left[-\frac{2M}{r}\right] = 0$$

$$\frac{\partial h_{ii}}{\partial x^1} = 0 \qquad \frac{\partial h_{ii}}{\partial x^\alpha} = -i\frac{2MX^\alpha}{r^3}$$

$$X^\alpha = -i x^\alpha$$

$$\boxed{\frac{\partial h_{ii}}{\partial x^\alpha} = -\frac{2Mx^\alpha}{r^3}}$$

$$\frac{\partial h_{mk}}{\partial x^m} = ?$$

$$\frac{\partial h_{m1}}{\partial x^m} = \underbrace{\frac{\partial h_{11}}{\partial x^1}}_{=0} + \underbrace{\frac{\partial h_{21}}{\partial x^2}}_{=0} + \underbrace{\frac{\partial h_{31}}{\partial x^3}}_{=0} + \underbrace{\frac{\partial h_{41}}{\partial x^4}}_{=0}$$

$$\frac{\partial h_{m\alpha}}{\partial x^m} = \frac{\partial h_{1\alpha}}{\partial x^1} + \frac{\partial h_{2\alpha}}{\partial x^2} + \frac{\partial h_{3\alpha}}{\partial x^3} + \frac{\partial h_{4\alpha}}{\partial x^4}$$

$$\frac{\partial h_{m\alpha}}{\partial x^m} = \frac{\partial h_{[\alpha]\alpha}}{\partial x^{[\alpha]}} = + \frac{M}{r^2}\frac{\partial r}{\partial x^\alpha}$$

$$\frac{\partial}{\partial x^\alpha} = -i\frac{\partial}{\partial X^\alpha} = \cancel{i}$$

$$\frac{\partial h_{m\alpha}}{\partial x^m} = -i\frac{M}{r^2}\frac{\partial r}{\partial X^\alpha} = -i\frac{M X^\alpha}{r^3}$$

$$\frac{\partial h_{m\alpha}}{\partial x^m} = -i\frac{M[-i\bar{x}^\alpha]}{r^3}$$

$$\boxed{\frac{\partial h_{m\alpha}}{\partial x^m} = -\frac{M x^\alpha}{r^3}}$$

$$\boxed{\frac{\partial h_{ii}}{\partial x^\alpha} = -\frac{2 M x^\alpha}{r^3}} \qquad \boxed{\frac{\partial h_{ii}}{\partial x^1} = 0}$$

$$\boxed{\frac{\partial h_{m\alpha}}{\partial x^m} = -\frac{M x^\alpha}{r^3}} \qquad \boxed{\frac{\partial h_{m1}}{\partial x^m} = 0}$$

$$\cancel{-2\frac{\partial h_{ii}}{\partial x^\alpha} + \partial h}$$

$$\frac{\partial h_{m1}}{\partial x^m} = \underbrace{\frac{\partial h_{11}}{\partial x^1}}_{0} + \underbrace{\frac{\partial h_{21}}{\partial x^2}}_{0} + \underbrace{\frac{\partial h_{31}}{\partial x^3}}_{0} + \underbrace{\frac{\partial h_{41}}{\partial x^4}}_{0}$$

h_{11} no depende del tiempo.

$$\boxed{B_{ij,k} = \frac{\partial h_{ki}}{\partial x^j} - \frac{\partial h_{ij}}{\partial x^k}} \quad (1)$$

$$\boxed{\frac{d^2 x^m}{ds^2} = \Delta^{mk}\left(\frac{\partial h_{ki}}{\partial x^j} - \frac{\partial h_{ij}}{\partial x^k}\right)\frac{dx^i}{ds}\frac{dx^j}{ds}} \quad (2)$$

$$\boxed{B^m_{ij} = \Delta^{mk} B_{ij,k}} \quad (3)$$

$$B^1_{11} = \Delta^{11} B_{11,1} = \frac{\partial h_{11}}{\partial t} - \frac{\partial h_{11}}{\partial x^1} = 0$$

$$\boxed{B^1_{11} = 0}$$

$$B_{11,1} = 0$$

$$B_{11,\alpha} = \frac{\partial h_{\alpha 1}}{\partial t} - \frac{\partial h_{11}}{\partial x^\alpha}$$

$$B_{1\alpha,1} = \frac{\partial h_{11}}{\partial x^\alpha} - \frac{\partial h_{1\alpha}}{\partial t}$$

$$B_{\alpha 1,1} = \frac{\partial h_{1\alpha}}{\partial t} - \frac{\partial h_{\alpha 1}}{\partial t} = 0$$

$$B_{1\alpha,\beta} = \frac{\partial h_{\beta 1}}{\partial x^\alpha} - \frac{\partial h_{1\alpha}}{\partial x^\beta}$$

$$B_{\alpha 1,\beta} = \frac{\partial h_{\beta\alpha}}{\partial t} - \frac{\partial h_{\alpha 1}}{\partial x^\beta}$$

$$B_{\alpha\beta,1} = \frac{\partial h_{1\alpha}}{\partial x^\beta} - \frac{\partial h_{\alpha\beta}}{\partial t}$$

$$B_{\alpha\beta,\gamma} = \frac{\partial h_{\gamma\alpha}}{\partial x^\beta} - \frac{\partial h_{\alpha\beta}}{\partial x^\gamma}$$

Tabla de símbolos de Birkhoff de segunda especie.

$$B^{1}_{11} = 0$$

$$B^{\alpha}_{11} = \frac{\partial h_{11}}{\partial x^{\alpha}} - \frac{\partial h_{1\alpha}}{\partial t}$$

$$B^{1}_{1\alpha} = \frac{\partial h_{11}}{\partial x^{\alpha}} - \frac{\partial h_{1\alpha}}{\partial t}.$$

$$B^{1}_{\alpha 1} = 0$$

$$B^{\beta}_{1\alpha} = \frac{\partial h_{1\alpha}}{\partial x^{\beta}} - \frac{\partial h_{1\beta}}{\partial x^{\alpha}}$$

$$B^{\beta}_{\alpha 1} = \frac{\partial h_{1\alpha}}{\partial x^{\beta}} - \frac{\partial h_{\alpha\beta}}{\partial t}.$$

$$B^{1}_{\alpha\beta} = \frac{\partial h_{1\alpha}}{\partial x^{\beta}} - \frac{\partial h_{\alpha\beta}}{\partial t}$$

$$B^{\gamma}_{\alpha\beta} = \frac{\partial h_{\alpha\beta}}{\partial x^{\gamma}} - \frac{\partial h_{\alpha\gamma}}{\partial x^{\beta}}$$

...ca (pseudounidimensional) ojo

Tabla de símbolos de Birkhoff de segunda especie.

$\boxed{\text{Campo central.}}$

$\boxed{\frac{\partial}{\partial t} = 0}$ $\boxed{h_{11} = h_{22} = h_{33} = h_{44} = \frac{M}{r}}$

~~∂h_{ij}~~ $h_{ij} = 0$ para $i \neq j$

$B^1_{11} = 0$

$B^\alpha_{11} = -\frac{Mx^\alpha}{r^3}$

$B^1_{1\alpha} = -\frac{Mx^\alpha}{r^3}$

$B^1_{\alpha 1} = 0$

$B^\beta_{1\alpha} = 0$

$B^\beta_{\alpha 1} = 0$

$B^1_{\alpha\beta} = 0$

$B^\gamma_{\alpha\beta} = ?$

$B^2_{\alpha 2} = B^3_{\alpha 3} = B^4_{\alpha 4} = 0$

Campo Central

Símbolos de Birkhoff de segunda especie

$B^1_{11} = 0$	$B^1_{12} = -\frac{Mx}{r^3}$	$B^1_{13} = -\frac{My}{r^3}$	$B^1_{14} = -\frac{Mz}{r^3}$
$B^1_{21} = 0$	$B^1_{22} = 0$	$B^1_{23} = 0$	$B^1_{24} = 0$
$B^1_{31} = 0$	$B^1_{32} = 0$	$B^1_{33} = 0$	$B^1_{34} = 0$
$B^1_{41} = 0$	$B^1_{42} = 0$	$B^1_{43} = 0$	$B^1_{44} = 0$
$B^2_{11} = -\frac{Mx}{r^3}$	$B^2_{12} = 0$	$B^2_{13} = 0$	$B^2_{14} = 0$
$B^2_{21} = 0$	$B^2_{22} = 0$	$B^2_{23} = +\frac{My}{r^3}$	$B^2_{24} = +\frac{Mz}{r^3}$
$B^2_{31} = 0$	$B^2_{32} = 0$	$B^2_{33} = -\frac{Mx}{r^3}$	$B^2_{34} = 0$
$B^2_{41} = 0$	$B^2_{42} = 0$	$B^2_{43} = 0$	$B^2_{44} = -\frac{Mx}{r^3}$
$B^3_{11} = -\frac{My}{r^3}$	$B^3_{12} = 0$	$B^3_{13} = 0$	$B^3_{14} = 0$
$B^3_{21} = 0$	$B^3_{22} = -\frac{My}{r^3}$	$B^3_{23} = 0$	$B^3_{24} = 0$
$B^3_{31} = 0$	$B^3_{32} = +\frac{Mx}{r^3}$	$B^3_{33} = 0$	$B^3_{34} = +\frac{Mz}{r^3}$
$B^3_{41} = 0$	$B^3_{42} = 0$	$B^3_{43} = 0$	$B^3_{44} = -\frac{My}{r^3}$
$B^4_{11} = -\frac{Mz}{r^3}$	$B^4_{12} = 0$	$B^4_{13} = 0$	$B^4_{14} = 0$
$B^4_{21} = 0$	$B^4_{22} = -\frac{Mz}{r^3}$	$B^4_{23} = 0$	$B^4_{24} = 0$
$B^4_{31} = 0$	$B^4_{32} = 0$	$B^4_{33} = -\frac{Mz}{r^3}$	$B^4_{34} = 0$
$B^4_{41} = 0$	$B^4_{42} = +\frac{Mx}{r^3}$	$B^4_{43} = +\frac{My}{r^3}$	$B^4_{44} = 0$

$$g_{ij,k} = \frac{\partial g_{ij}}{\partial x^k} - B^m_{ik}\, g_{mj} - B^m_{jk}\, g_{im} = 0$$

$$g_{11,1} = \frac{\partial g_{11}}{\partial t} - B^m_{11}\, g_{m1} - B^m_{11}\, g_{1m} = 0$$

$$B^m_{11}\, g_{m1} = -\frac{Mx}{r^3} g_{21} - \frac{My}{r^3} g_{31} - \frac{Mz}{r^3} g_{41}$$

$$\boxed{\frac{\partial g_{11}}{\partial t} = - \frac{2M}{r^3}\left[x g_{12} + y g_{13} + z g_{14}\right]}$$

$$g_{11,2} = \frac{\partial g_{11}}{\partial x} - B^m_{12}\, g_{m1} - B^m_{12}\, g_{1m} = 0$$

$$B^m_{12}\, g_{m1} = -\frac{Mx}{r^3} g_{11}$$

$$\boxed{\frac{\partial g_{11}}{\partial x} = -\frac{2Mx}{r^3} g_{11}}$$

$\cancel{g_{11}}$

$$g_{11,3} = \frac{\partial g_{11}}{\partial y} - B^{m}_{13}\, g_{m1} - B^{m}_{13}\, g_{1m} = 0$$

$$\boxed{B^{m}_{13}\, g_{m1} = -\frac{My}{r^3} g_{11}}$$

$$\boxed{\frac{\partial g_{11}}{\partial y} = -\frac{2My}{r^3} g_{11}}$$

$$g_{11,4} = \frac{\partial g_{11}}{\partial z} - B^{m}_{14}\, g_{m1} - B^{m}_{14}\, g_{1m} = 0$$

$$\cancel{\frac{\partial g_{11}}{\partial z} = -2}$$

$$B^{m}_{14}\, g_{m1} = -\frac{Mz}{r^3} g_{11}$$

$$\boxed{\frac{\partial g_{11}}{\partial z} = -\frac{2Mz}{r^3} g_{11}.}$$

$$g_{12,1} = \frac{\partial g_{12}}{\partial t} - B^{m}_{11} g_{m2} - B^{m}_{21}$$